The Usefulness of the Impossible

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A straight line has no width, no depth, no wiggles, and no ends.

There are no straight lines. We have ideas about these non-existent impossibilities: we even draw pictures of them. But they do not exist.

Ask a draftsman to draw a straight line. Place his product under a microscope. Observe the variable width; the darkness and lightness which mark its varying depth; the wiggles, both lateral and vertical, where the line plunges and climbs and wriggles among the fibers. Put the microscope away and observe the ends of the line where it runs off the paper. Finally, contemplate the curvature of space.

A straight line hasn't even a definition. Heath, in his *Thirteen Books of Euclid's Elements*, discusses the problem. Before Euclid, Parmenides has stated "straight is whatever has its middle in front of both ends." Euclid defined a straight line as "a line which lies evenly with the points on itself." Heron, in the first century, A.D., suggested "a line stretched to the utmost." Equally old, although restated by Leibniz and put into the following form by Gauss, is: "The line in which lie all points that, during the revolution of a part of space about two fixed points, maintain their position unchanged...." While this may seem definitions aplenty, the modern view as expressed by Pfleiderer is: "It seems as though the notion of a straight line, owing to its simplicity, cannot be explained by any regular definition which does not introduce words already containing in themselves, by implication, the notion to be defined, as though it were impossible, if a person does not already know what the term straight here means, to teach it to him unless by putting before him in some way a picture or a drawing of it."

A point has no dimensions, no existence, and no definition. A picture of a point is a ragged area of uncertain extent on a rough surface. By refining the picture, the area diminishes. Finally, we can refine no more. We place the portraits side by side in the sequence of successive refinement. Then we point far to the side

and say, "The picture that belongs there, where refinement has been carried to the ultimate and the dimensions have entirely vanished; that picture, if it existed, would be a true dimensionless point."

Euclid lists twenty-three definitions which define more than twenty-three figments of the imagination. Next he postulates an ability to draw straight lines from point to point, to project these straight lines indefinitely in either direction, and to draw circles; all manifest impossibilities. He assumes all right angles are equal, although there are no right angles. And he includes the famous postulate of parallels, by denying which Riemann created the geometry of curved space. Lastly, Euclid introduces five "common notions" as axioms; that is, as self-evident truths; the very first of which is impossible, let alone true. "Things equal to the same thing are equal to each other." No two real things are precisely equal. The common equal is doubly doubtful. It is an impossible ideal which can be approached only imaginatively.

He who protests that this is a quibble, that for practical purposes equals do exist, merely impales himself on the other horn of a dilemma. In a chapter on Number in his *Aspects of Science*, Tobias Dantzig tells of two bars, A and B, so nearly the same length as to defeat all attempts to ascertain which is the longer. Practically, they are of equal length. Another bar, C, is so nearly the same length as B as to defy all attempts to show a difference. Practically, B and C are also of equal length. But when A is compared with C, there is no difficulty in proving that C is longer than A. So if we deal with practical equalities, things may be equal to the same thing without being equal to each other.

The whole of geometry is consciously, wilfully, deliberately antagonistic to reality. In classical geometry, the compass and straight edge are allowed, the ruler forbidden. The compass and straight edge are both mystical, for they produce true circles and lines. The ruler is a practical tool used by artisans and beneath the dignity of a Greek philosopher-mathematician. Modern geometry has exceeded the purity of the ancients. It deals with points which are not points but vague unspecified items; lines which are not lines but classes of items; and planes that are not planes but classes of classes. In some modern geometries, straight lines are distorted geodesics twisting and wriggling in a warped and changing space. In modern physics, these writhing monsters are chopped into a large, but finite, number of tiny, but not infinitesimal, discontinuous, discrete quanta. This is as near as we can get to "real" straight lines!

Nor is geometry the black sheep of mathematics. All mathematics carries the family taint; all mathematics is a gigantic tussle with nonexistent impossibilities. We are cautioned not to add poems and railroad trains, or to subtract centimeters from miles. Without referring to what is added on the balance sheets of business enterprises, let us merely note that if one ought not to add apples and bananas, one probably shouldn't add Jonathans and Rome Beauties, or big apples and little ones, or 1929 dollars and 1932 dollars. If we are to add at all, we must add unlikes, in violation of all mathematical regulations.

I shall not attempt to prove that mathematics is useful. I will admit it and so save myself the trouble of proving that here is a great and respected discipline where all is impossible and yet much is useful. The usefulness largely flows from the impossibility. Mathematical concepts have been simplified and generalized until they describe an imaginative world no part of which could possibly exist outside men's minds. But their simplicity and generality have made them amenable to the laws of logic. We can think about them with sufficient rigor to build a truly impressive edifice, much of which translates into physics and engineering.

II

Truth to the mathematician merely means freedom from internal inconsistencies. All mathematics begins with a set of axioms. Any set of axioms is as valid as any other as long as it avoids contradictory assumptions. Physics supposedly labors under the additional handicap of the experimental method. Its assumptions must be consistent with the readings of its meters and its gauges. The superstructure based on these assumptions must submit to experimental verification. As a result, the novice believes physics describes objective reality. Only mathematics enjoys a greater reputation for the profundity and pervasiveness of its Truths.

Physics, too, is plagued by questionable tactics. Laws proved untrue are easily rescued by adding terms. For example, Boyle's law was found untrue for high pressures and low temperatures. Van der Waal argued that it held precisely only for a perfect gas with point molecules; that as the molecules of a real gas crowded closer together the error became more noticeable. He saved Boyle's law by introducing another term to take care of the size of the molecules. If more refined experiments reveal further discrepancies, the law may be rescued again by introducing corrections for the velocities of the molecules, or for their nationalities. If the

velocities won't explain the hypothetical inaccuracies of the law, the physicist may try accelerations or differential equations of still another order. If a proportional law doesn't fit, he can try inverse proportions, or squares, or exponents. Somewhere he can find a physical measurement which seems to have some kind of mathematical relation to the observed deviations, and all such troubles will surely yield to the same treatment.[1]

Newton's laws of dynamics exhibit a refinement of the technique. Nowhere has anyone ever seen a body continue moving in a straight line with uniform velocity. Nor has anyone ever seen a body at rest remain at rest. Indeed, we do not even know what the words "at rest" mean. I quote Dantzig loosely and out of context: "How can a bird fly in a straight line and at constant speed in the teeth of gravity? The answer is that the *resistance* of the air balances the gravitational pull. How can a ball roll down an inclined path at constant velocity instead of constantly accelerating? The friction of the surface accounts for this. Why do the particles of a solid body stay put, instead of flying asunder under the action of gravity? *Cohesive internal forces* keep them together. Whenever and wherever a violation of the principle of inertia is observed, it is sufficient to invoke some reaction to have the difficulty vanish, as though by magic." The accountant has a nasty name for the technique. He calls it "plugging" to force a balance.

From plugging the accounts, it is only a short step to the next refinement. The physicist avoids the need of a rescuing plug by making his laws true by definition. I quote Poincaré's great work, *Science and Hypothesis:* "The principles of dynamics at first appeared to us as experimental truths; but we have been obliged to use them as definitions. It is by definition that force is equal to the product of mass by acceleration; here, then, is a principle which is henceforth beyond the reach of any further experiment. It is in the same way by definition that action is equal to reaction." Having set up his definitions, the physicist calibrates his instruments accordingly. Having defined force as proportional to acceleration, and having chosen some force as a unit, he doubles the acceleration and marks his force meter two at the point indicated. Ever after, whenever he measures with the instruments so created, his findings bear out his definitions; his laws become absolutely true.

The culmination of the technique is the creation of so anthropomorphic a cosmology as to be beyond the ability of men to prove or disprove it. Eddington states the case thus: "We have found a strange footprint on the shores of the unknown. We have

[1] Physicists tell me that this paragraph isn't fair.

devised profound theories, one after another, to account for its origin. At last we have succeeded in reconstructing the creature that made the footprint. And lo! it is our own." Dantzig goes farther. I quote at length:

"For however phantastic a universe our mind may conceive, our mind can also conceive it peopled by species, endowed with consciousness, intelligence and mobility, which in the course of time would arrive at a cosmology identical with our own.

"Seeking permanence in the shifting chaos of their perceptions, these beings would eventually discover in their environment bodies which would behave in relative unison to their own. Accepting these bodies for rigid standards, they would proceed to survey and measure the universe with their aid. Singling out some cyclic phenomena which recur in relative synchrony to each other, and to their own physiological processes, these beings would finish by identifying these temporal series with their own stream of consciousness. Convinced that their universe was independent of their consciousness, they would affirm the objective character of their conception of time, and proceeding beyond the narrow confines of their own experience, they would extend their conception to the world at large, conceiving the latter as floating with absolute uniformity on the stream of duration. And transferring to their universe their own physiological and psychological attributes, they would fill space with bristling forces and shackle history to a causal chain."

As with mathematics, I propose to assume that physics is useful, although I feel some doubt has been cast upon its objective validity. The rigorous exclusion of all nonmeasurable phenomena, and the careful formulation of its definitions and axioms as the calibrations of the instruments to be used in the experimental verification of physical laws, have simplified and generalized physics along the lines of the mathematical model. This has added immensely to its precision, to its power, and to its usefulness.

III

Mathematics and physics are theoretical. Let us turn from abstraction and generalization to practical application. Engineering is as riddled with impossibility as mathematics or physics. Of course, engineering is full of mathematics and physics; they are the basic sciences. But I do not rest my case here. Engineering data are as impossible as engineering's mathematical method. Engineering data are average values, usually treated in engineer-

ing calculations as absolutes. According to Mills' *Materials of Construction*, structural steel has an elastic limit of 35,000 pounds per square inch, a tensile strength of 65,000 pounds per square inch, and a modulus of elasticity of 30,000,000 pounds per square inch. No standard deviations are given. Such are found only in the inexact, semi-scientific disciplines of biology, psychology and economics.[1]

In calculating the distortion of bridge members, the engineer implicitly assumes constant cross-sections, uniform crystal structures, and homogeneous chemical composition from end to end of each beam. Anyone who has seen the scale peel off an ingot as it goes through the rolls knows the constant cross-section is a crude fiction. Heat treatment and the working of steel so change crystal structures as to make the assumption of uniformity in ordinary rolled beams heroic indeed.

However, the engineer is a practical fellow. While his equations assume a 35,000-pound elastic limit in a perfectly uniform beam, he does not. To keep his bridges from falling when these assumptions err on the wrong side, he typically designs them to carry seven times the expected maximum load. This makes bridges expensive but safe. The engineer can boast that they seldom fall. Yet engineers are modest braggarts. The multiplier used to assure safety has been rechristened the "factor of ignorance."[2]

IV

In mathematics, physics, and engineering, we see that the impossible may be useful—at least as a calculating device. My purpose so far is not to pillory these respectable and useful disciplines, but rather to throw a mantle of respectability over the shady dealings I am about to perpetrate. I propose to apply the method of the impossible calculating device to the science of economics. As in mathematics, we begin with a few impossible assumptions; go where we can; and introduce further impossible assumptions whenever we get stuck.

Let us assume all men are alike; are exclusively gain motivated; know all the facts; always behave logically; act as completely

[1] This is true only of elementary works.

[2] Although the engineer claims to *know*, and patronizingly refers to business judgments as wild, inaccurate guesses, the facts do not seem to bear out his claims to precision. Business men seem to be almost exactly as ignorant as engineers. Many business men will not make improvements unless they believe savings of the first two years will return the original investment. If we assume 7% is a reasonable return on the owner's investment, this provides a "factor of ignorance" of almost precisely seven.

independent social atoms; and can transfer from job to job or place to place instantly and without loss or regret. There are no such men.

Let us assume that such economic men live in a world of diminishing returns and have their sole intercourse in free markets. A free market, by definition, involves private property; no collusion among buyers or sellers; instantaneous complete communication among all participants; and no coercion—every trade must be wholly voluntary on both sides. There are no such markets.

These assumptions may be redundant; they may even be in conflict—surely the legal part of my audience will permit such economic license. Probably such an economic world, so peopled, would disintegrate through self-generated centrifugal tendencies. There is no religion and no tradition to hold them together. There is assumed to be a state which enforces contracts, prevents stealing and cheating, and preserves the peace of the realm.

Still seeking simplicity, we create an imaginary world for these imaginary people; a new continent in which every square foot of surface has the same fertility, drainage, temperature, rainfall and sunlight; following the same unvarying pattern year after year.

Let us place one of our standard economic men on our new continent. If he farms a single square inch, his product will be nil. If he attempts to farm the whole continent, he will be too busy walking to plant, cultivate, or harvest. Somewhere between these limits his product reaches a maximum—the goal of the economic man. We will call this optimum area a plot, and will assume exactly a thousand such plots on the continent. We will further assume that one man can produce 80 tons of product per year on one such plot. If he cultivates more or less land, his annual product will be less than 80 tons. By the law of diminishing returns, we know that he can produce more than 40 tons on half a plot, for he can weed, cultivate, and irrigate more intensively. Let us assume that on half a plot he could produce 70 tons and on a third of a plot 60 tons.

Let us add economic men. Each takes possession of a plot and reaps a harvest of 80 tons annually until we reach a population of a thousand. Then all the plots have been pre-empted; the West has been won; the frontier is closed. We add the thousand and first man and ominous social changes take place. He becomes the first of a landless wage-earning class. Since two men on one plot, or one on each of two half-plots, can produce 140 tons, the newcomer adds 60 tons to the product; and that is the maximum wage any land owner will pay. The free market and competition among

land owners will drive the wage offered up to, but never beyond, 60 tons.

Now we add more economic men to our mythical continent. No significant economic change occurs until the two thousand and first man arrives. He must cut his price to take a job away from one of the earlier immigrants. But there will always remain one man unemployed and underquoting the prevailing wage rate until it is hammered down to 40 tons. Since three men on a plot can produce 180 tons, the third man adds 40 tons to the gross income of the landowner and can be hired at that rate without firing his predecessor.

Let us add a few more economic men to get safely beyond the critical number and then review and take stock. If we add a man, the total social product increases 40 tons, as does the gross income of the owner of the land on which he works; and this fixes the wage rate, for no owner will give more and competition among owners makes it unnecessary for the laborer to take less. If a landowner retires, his gross income falls from 180 to 140 tons—evidently he gets 40 tons for his own work. Having retired, he gets a gross income of 140 and pays each of two men 40 tons, leaving 60 for himself. This, then, is the earning power, or rent, of the land. Suppose we add a plot to the continent. Then a man will leave each of two plots on which three men have been working to take up the new land. Each of the plots they leave produces 40 tons less per year. But the new plot produces 140 tons, a net gain of 60 evidently imputable to the land rather than to the labor. Thus whether we add or subtract land or laborers, we find that the wage rate comes out 40 and the rent 60. The ratio of land to labor fixes both wages and rent.

V

Now we add a spellbinder. He promises more *Lebensraum* and a higher standard of living for the Master Race. He is elected *Führer* and begins to make good on his promises. By blaming Jews for the desperately low wages, he starts a pogrom, which reduces the pressure of population on the soil, and thus tends to raise the wages of the survivors. He adds to the *Lebensraum* by conquering neighboring lands, but he must take them relatively empty or the population pressure will continue unaltered. He bombs civilians, ostensibly to break morale and win a war. He relocates whole populations to make peoples conform to boundaries. He dispossesses the private owners of conquered lands to give the

Master Race the product imputed by the market to land as well as the product of their own toil.

Or the champion of the propertyless masses can employ emergency relief and boondoggling make-work schemes to draw labor from the competitive market until the marginal productivity, and the wage level, of privately employed labor is raised to any desired standard. This will be expensive, but an unbalanced national budget will temporarily finance any requirements of the program which cannot be met by soaking the rich.

A union leader's powers are more circumscribed. He, *der Führer*, and the Great Democratic Leader could raise living standards by a program for increasing the productivity of labor. This way to a better life requires thoughtful intelligence and, consequently, seldom appeals to the purveyors of emotion whom we perennially elect as dictators, presidents, and union leaders. The path of restricted supply requires less effort and intelligence, usually resting on force, which such people understand better. To the union boss, this means a closed shop, enforced by strikes and picketing. But the closed shop will not reduce the ratio of labor to other factors of production unless membership in the union is sharply restricted. This is usually accomplished by high dues and initiation fees, or by arduous apprenticeship requirements, preferably the former, as these enable the maintenance of elaborate country estates. All methods of restricting entrance to a trade are anti-social, as they push labor from the more remunerative vocations into the relatively overcrowded nonunion occupations. However, the union members benefit by higher wages and cheerfully re-elect their business manager. The exploited consumer and the depressed labor in nonunion occupations have no vote in union elections.

Real leaders in a real world may discover other opportunities for raising wages. The premises of our theory do not accurately describe the real world. Consequently, real wages may fall below the marginal productivity of labor, the wage rate for our impossible world of economic freedom. Laborers are not economic men. They do not know all the facts. Unions or governments may provide information about opportunities in other companies and towns. Laborers cannot transfer easily from job to job or place to place without loss or regret. Moving costs money, and union or government may advance money to finance such moves. Moving breaks social ties, and the union may facilitate entry to the social life of the new community. Nor do laborers think logically. The employee-employer relationship involves much besides wages

and hours; much which individual laborers are prone to overlook. So government and union have a role to play in settling secondary matters, such as safety provisions, conditions of work, vacations, seniority, and many more.

The market is not always coercion-free as assumed. A single large company operating in a small company town is likely to be a monopsony. Trade associations may rig the labor market. Government and union may go in for trust busting; or the union may organize supply to confront monopsony with monopoly.

VI

Perhaps it will be contended that I have not demonstrated the usefulness of the impossible; that I have arrived at the same self-evident conclusions everyone holds without recourse to such theoretical analysis. I do not believe so. Laborers and unions have long been committed to the lump-of-work fallacy. They have been fearful of doing too much, of working too fast, and so causing unemployment. This restriction of output makes men less valuable to employers and society, and so depresses wages. It is disadvantageous for employer, employee and community. The restriction on entrance to a trade undoubtedly raises wages in the restricted field at the cost of depressing wages elsewhere. Those damaged by exclusion have concentrated their efforts on establishing restricted monopolistic fields of their own. Nowhere have the disinherited fought for their American heritage—the right to choose their vocation.

Unions have blindly fought for higher wage rates without considering consequences. They have assumed that a certain lump of work had to be performed regardless of the price charged for it. The building trades in particular have been successful; so successful that there was almost no building construction for ten years during the Great Depression of the 1930's—surely a potent demonstration that buyers buy less at high prices than at lower. A war demand with government financing surmounted the barrier of high construction costs, and factory building boomed. Impending inflation made bonds, insurance, and money-hoarding unsatisfactory vehicles for savings. The shadow of new taxes made corporate securities uncertain and undesirable. What was one to do with his money, if any? In spite of construction costs, the accumulating housing shortage, together with the ominous threat of inflation and the hazardous nature of every other form of investment, have produced a residential building boom.

Yet much of our last unlamented Great Depression was due to absurd costs of construction. A large part of our unemployment was in the building trades, which were almost altogether unemployed. Hourly wages in these occupations were higher in 1934 than in 1929, although practically all other prices, particularly rents, were from 25% to 50% lower. Let us return to our impossible calculating devices to seek an explanation for such mad behavior.

By our assumption of diminishing utility, buyers will be ready to buy more buildings if their prices fall. Given one big industrial union, a wage reduction would result in a comparable reduction in building costs, which would result in substantially more construction and employment. In general, labor will gain more in employment than it loses in lowered hourly rates up to the point at which the market is cleared and unemployment vanishes. The industrial union could learn this lesson by bitter experience and the mad spiral of rising costs and declining construction would cease.

We have trade unions in the building industry, and a trade union cannot learn this lesson, because it doesn't apply to a trade union. If any union, acting independently, reduces its wage rate, total building costs are negligibly reduced and building volume insignificantly increased. The union members gain little in increased employment and lose much in lowered wage rates. Meantime, all the other unions benefit equally by the increased building activity without reducing their rates.

Conversely, if any one trade raises its rate, building costs increase a little and construction activity decreases a little. All building labor shares the decrease in employment, but one union alone reaps the advantage of a higher wage rate. Consequently, they gain more in the increased rate than they lose in the decreased employment. Under the assumed conditions, if each union acts independently, they will take turns raising their rates. The raises will cumulate and the total impact on building activity will put an end to the industry; labor will have committed economic suicide. If the unions should coalesce into one big industrial union, effect would follow cause promptly and simply. The leaders and the members might learn and the industry might not throw more than a million men on relief. Theory supports the C.I.O. against the A.F. of L. in the building trades unless the inter-union councils gain power and finally become the bargaining agent for all member unions acting in unison.

VII

I will conclude with one last example of the usefulness of the impossible in avoiding derailment of a train of analysis. It will show why a man may successfully operate a business and still hold dangerous political ideas. A successful budgeteer friend of mine believes what he sees and will have no truck with the impossible. He believes in induction to the total exclusion of deduction. His ideas of the causes and cure of depressions are simple: All industries which have organized to agree on prices and control their markets are generally profitable. The industries which follow the old tradition of free competition are in chaotic condition and are chronically depressed. To achieve universal prosperity, we should compel the competitors to organize, to cease cutting each others' throats. This was, of course, the NRA program.

We have already seen how labor monopolists gain advantage at the expense of unorganized consumers and laborers; how restricted entrance maintains wages in one field by further depressing wages elsewhere. Universal organization prevents any group from achieving prosperity by monopoly because it would leave no one to exploit. All market rigging by unions and by industries involves restricted outputs. Normally the organized steel industry seldom operates above 60% or 70% of capacity and makes a profit at any rate above 45%. The unorganized agricultural industries operate near 100% all the time and are chronically depressed. I say the steel industry has behaved anti-socially, and should be subjected to enough policing to enforce competition. If this proves impossible, the steel industry should be socialized. On the other hand, agriculture has done its social best. Aside from conservation measures, it should be let alone.

Suppose all industries are forced to disorganize, to allow markets to be free. Then all would operate at capacity, and prices would be cut until the market was cleared. The maximum production would be maintained and consumed; the standard of living would reach the highest possible level.

Suppose all industries follow my friend's advice and organize to control their markets, to bring order out of chaos. Then all restrict outputs, all operate well below capacity. The total product is much less, the population remains the same, and the standard of living must be substantially lowered.

We have seen how useful the impossible has become in mathematics, physics, and engineering. If the same simplified, generalized calculating technique be applied to economics, the solutions

to many real problems will be sharp and clear. Perhaps this will make the appeal to emotion unpopular and unprofitable. Then we may have done with artificial shortages imposed by such government schemes as NRA and AAA, by monopolistic combines acting in collusion to restrict output and rig markets, and by unions restricting entrance and murdering industries.